NOTATION

Nu	is the Nusselt number;
T_m	is the characteristic temperature;
T_w	is the filament temperature;
T_∞	is the stream temperature;
V	is the voltage drop on the probe filament;
R_w	is the resistance of the hot filament;
l	is the length of the etched section of the probe filament;
L	is the distance between filament holders;
d	is the diameter;
λ_1	is the coefficient of thermal conductivity of air at the characteristic temperature T_m;
ΔT	is the superheat of the filament relative to the stream temperature;
U	is the velocity of the oncoming stream;
ν_m	is the coefficient of kinematic viscosity of air at the characteristic temperature T_m;
Re_w	is the Reynolds number;
$\sqrt{U'^2}$	is the rms value of pulsations of the longitudinal velocity component.

LITERATURE CITED

1. D. C. Collis and M. J. Williams, J. Fluid Mech., 6, Part 3 (1959).
2. E. U. Repik and V. S. Ponomareva, Izv. Akad. Nauk SSSR, Sib. Otd., Ser. Tekh. Nauk, Part 3, No. 13 (1969).
3. G. E. Andrews, D. Bradley, and G. F. Hundy, Int. J. Heat Mass Transfer, 15, 1765 (1972).
4. B. Gebhart and L. Pera, J. Fluid Mech., 45, 49-64 (1971).
5. G. Comte-Bellot, A. Strohl, and E. Alcaraz, J. Appl. Mech., No. 4, 767 (1971).

EMPIRICAL HEAT-EXCHANGE EQUATION FOR A HYDRORESISTOR THERMOANEMOMETER

V. N. Nabatov

UDC 53.082.6

An empirical equation is obtained which well describes the heat-exchange process at the sensor of a hydroresistor thermoanemometer in the range of Reynolds numbers from $3 \cdot 10^3$ to 10^5 and with a Prandtl number of 7.5.

The hydroresistor thermoanemometer is one of the principal instruments in the investigation of the microscale variability of the velocity field of oceanic currents [1, 2]. In its technical data it is little inferior to the best models of film thermoanemometers, while the high reliability under natural conditions has established its predominant role. Despite the fact that the properties of this measuring device are similar in many ways to the properties of an ordinary thermoanemometer, there are important differences in the construction of the sensor and the character of its heat exchange with the surrounding medium.

The proposed empirical equation describing the steady process of heat exchange in existing models of hydroresistor thermoanemometers is analogous to King's equation for a wire thermoanemometer [3]. It can be used to calculate the sensitivity to velocity pulsations in various modes of heating of the sensor.

The sensor of a hydroresistor thermoanemometer is represented schematically in Fig. 1. Cooling of a heated layer of electrically conducting liquid (seawater) at the surface of the head of the insulator 2 near the round microelectrode 1 occurs during longitudinal flow over it. The heating is accomplished by an alternating voltage applied between the microelectrode and the metallic holder 3. The frequency of the voltage is chosen so that the reactive component of the impedance of the sensor is much less than the active component:

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 5, pp. 827-833, November, 1978. Original article submitted December 13, 1977.

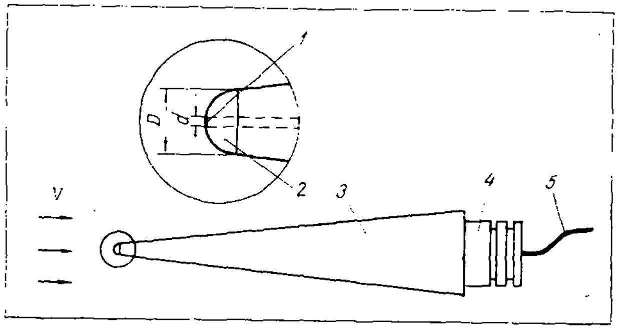

Fig. 1. Sensor of a hydroresistor thermoanemometer: 1) microelectrode; 2) head; 3) holder; 4) seat; 5) connecting cable.

$$R = A/\sigma, \tag{1}$$

where A is a constant of the sensor determined by the diameter d of the microelectrode and the geometry of the head (in the majority of cases it is found experimentally).

Henceforth for simplicity we will consider a hemispherical shape for the head with a diameter D. In the general case of an axisymmetric stream-lined head D is the diameter of a hemisphere equivalent with respect to the flow pattern near the critical point of the stream – the point where the microelectrode is located.

When the study voltage of the sensor is low, heating of the liquid hardly occurs and the effective electrical conductivity is equal to the electrical conductivity σ_0 of the surrounding medium at the temperature t_0, while the resistance of the sensor is the "cold" resistance:

$$R_0 = A/\sigma_0. \tag{2}$$

From the effective value of the electrical conductivity of the water in the averaging volume of the element and its known salinity one can find the effective heating temperature t and the effective superheat $t - t_0$ of the water. Then the heat-transfer coefficient is defined just as in the case of an ordinary thermoenemometer:

$$\alpha = I^2R/F(t - t_0). \tag{3}$$

The value of α is a function of the stream velocity, the parameters of the medium, and the geometry of the sensor. Usually this function is a consequence of an empirical criterial equation for the steady heat-exchange process. For example, Kramers [4] has suggested the following empirical equation for a wire thermoanemometer:

$$\mathrm{Nu} = 0.42\,\mathrm{Pr}^{0.2} + 0.57\,\mathrm{Pr}^{0.33}\,\mathrm{Re}^{0.5}. \tag{4}$$

The correctness of (4) was demonstrated in a range of Reynolds numbers of 10^{-2}-10^{5}. From this equation it is easy to obtain the well-known King equation.

A similar equation can also be obtained for a hydroresistor thermoanemometer, i.e., a connection can be found between the dimensionless heat-transfer coefficient – the Nusselt number – and the other criteria determining the heat exchange.

In the case of forced convection, one must include Re and Pr among the determining criteria for a hydroresistor thermoanemometer, as for a wire or film thermoanemometer. The diameter of the sensor head is taken as the characteristic size in these criteria. In addition, an auxiliary "geometrical" criterion d/D is introduced to allow for the influence on the heat exchange of the location of the heated section of liquid on the surface of the head.

In selected intervals of variation of the criteria, the functional connection between them can be represented in the form

$$\mathrm{Nu} = C\,\mathrm{Pr}^{l}\mathrm{Re}^{n}\,(d/D)^{m}. \tag{5}$$

Here C, l, n, and m are constants whose values can be obtained through an experimental test of this equation.

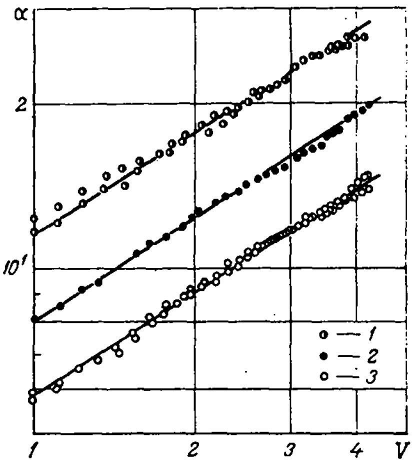

Fig. 2. Dependence of heat-transfer coefficient α ($W/cm^2 \cdot deg$) of heated layer of water on velocity V (m/sec) of oncoming stream for the following dimensions of the head of the sensor: 1) D = 0.3 cm; 2) 0.7; 3) 1.6.

Since the thermoanemometers are mounted on towed carriers in the majority of cases in ocean measurements, we will consider the flow of a stream with a velocity of more than 1 m/sec over the sensor, which corresponds to Reynolds numbers of about 10^4 for actual heads with diameters of 3-8 mm. It should be noted that at such values of Re the first term in (4) (on its right side) can be neglected in comparison with the second term, and then it has a form analogous to (5) (only the "geometrical" criterion is absent).

To determine α in an experimental test of Eq. (5) one must know the area F of cooling of the heated layer of liquid. In this connection we measured the distribution of water temperature over the surface of the head with longitudinal flow over a model of the sensor. The parameters of the model are D = 16 cm and d = 2 cm. The values of Re, Pr, and d/D were $3.2 \cdot 10^4$, 7.5, and 0.0125, respectively. To achieve similarity of the electromagnetic field we used one more criterion [6]: $d\omega\mu\sigma = \text{const}$.

The measurements were conducted with a microthermistor having an averaging scale of 0.5 mm. It was found that the water temperature at the surface of the head in a layer with a thickness of 0.5d has a maximum at the boundary of the microelectrode, at the point of the greatest electric field strength, and then declines and does not exceed 2% of the maximum value at distances of 3d from the center of the microelectrode. Thus, the heating area is determined mainly by the diameter d of the microelectrode, and it can be defined as the area of a circle with a radius of 3d, $F = 9\pi d^2$, for a small enough ratio d/D (with an error of less than 8% for $d/D \leq 0.13$).

The ratio of the maximum superheat of the water at the boundary of the microelectrode to the effective superheat calculated from the values of the "cold" and "hot" resistances at Re = $3.2 \cdot 10^4$ was about seven. This ratio grows with a decrease in the Reynolds number, so that when the maximum superheat is 30-40°C the effective superheat usually does not exceed 5°C. This fact makes it possible to use the values of the criteria at the temperature t_0 of the surrounding medium in the criterial equations.

The number of constants subject to experimental determination in (5) can be reduced. To be exact, in longitudinal flow over the sensor the center of the microelectrode coincides with the critical point of the oncoming stream. The total heat transfer of the element depends on the thickness of the hydrodynamic boundary layer and the distribution of the local velocity beyond its limits. In flow over a sphere the thickness of the boundary layer is proportional to $\sqrt{D/V}$ while the distribution of the local velocity near the microelectrode (near the critical point) can be represented in the form [7]

$$v = 3\frac{V}{D}l, \tag{6}$$

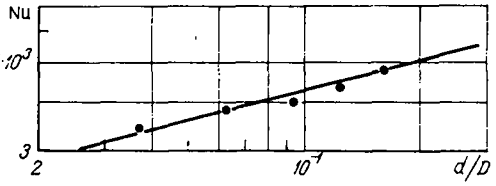

Fig. 3. Dependence of dimensionless heat-transfer coefficient Nu on the ratio d/D.

where l is the distance from the center of the microelectrode, with the range of its variation important for heat transfer depending on the diameter d of the microelectrode: $l_{\min} = d/2$ is the start of the heated layer and the boundary of the electrode; $l_{\max} = 3d$ is the boundary of the heated layer.

On the basis of what has been said, one can assume that when the properties of the liquid are constant the heat-transfer coefficient depends on the ratio V/D and the electrode diameter d, i.e.,

$$\alpha = f(V/D,\ d). \tag{7}$$

On the other hand, by substituting the expressions for the criteria Nu = $\alpha D/\lambda$ and Re = VD/ν into (5) we obtain

$$\alpha = C\,\mathrm{Pr}^l \frac{\lambda V^n}{\nu D^{1+m+n}}\, d. \tag{8}$$

From (7) and (8) it follows that m = 2n − 1 and Eq. (6) takes the form

$$\mathrm{Nu} = C\,\mathrm{Pr}^l\,\mathrm{Re}^n\,(d/D)^{2n-1}. \tag{9}$$

The value of the exponent n was obtained from the experimental dependence of the heat-transfer coefficient α on the velocity V of the oncoming stream. The measurements were made in a jet of seawater with a salinity of $35^0/_{00}$ and a temperature of 20°C on a hydraulic stand of the gravity type having a jet velocity which varies linearly with time from 1 to 4.2 m/sec. Three sensors with microelectrodes 0.4 mm in diameter and heads 3, 7, and 16 mm in diameter were investigated. The heating was done by a current with a frequency of 200 kHz. The range of Reynolds numbers was $3\cdot10^3$-$7\cdot10^4$ and the Prandtl number was 7.5, while the ratio d/D had three values: 0.13, 0.057, and 0.025. The error in the determination of α was about 10%.

The results of the measurements are presented in Fig. 2. The experimental points agree well with a power law of variation of α with an exponent n = 0.63:

$$\alpha = \mathrm{const}\cdot V^{0.63}. \tag{10}$$

It follows from (9) that with Re = const, Pr = const, D = const, and n = 0.63 the Nusselt number should depend on the diameter of the microelectrode:

$$\mathrm{Nu} = \mathrm{const}\cdot(d/D)^{0.26}. \tag{11}$$

The results of an experimental test of this equation on a sensor model with D = 16 cm and Re = $5.3\cdot10^4$ are presented in Fig. 3. The electrode diameters ranged from 6 to 26 mm. The measurements were conducted in a water flume with a water salinity of $4^0/_{00}$ and a temperature of 18°C, so that the Prandtl number remained the same as in the measurements of the dependence of α on V.

The following criterial dependence (Fig. 4) was obtained from the data of Fig. 2 as well as of measurements on the model with stream velocities of 20, 33, and 75 cm/sec in the water flume:

$$\mathrm{Nu} = 7.6\,\mathrm{Re}^{0.63}(d/D)^{0.26}. \tag{12}$$

The value of the Prandtl number enters into the coefficient 7.6 here. It was not possible to find the dependence on Pr, i.e., to obtain a fully universal equation of the type of (9), because of the insufficient accuracy in the determination of the values of the criteria. The exponent l for Pr in Eq. (9) should be expected to be close to 0.3-0.4 by analogy with (4). In this case variation in the Prandtl number by 30% leads to variations in Nu by 10%, equal to the measurement error. Under our conditions it was difficult to obtain a large range of variation of Pr and increase the measurement accuracy.

The empirical equation (12) obtained is suggested for describing the steady process of heat exchange of at the sensor of a hydroresistor thermoanemometer in the range of Reynolds numbers of $3\cdot10^3$-10^5, the range

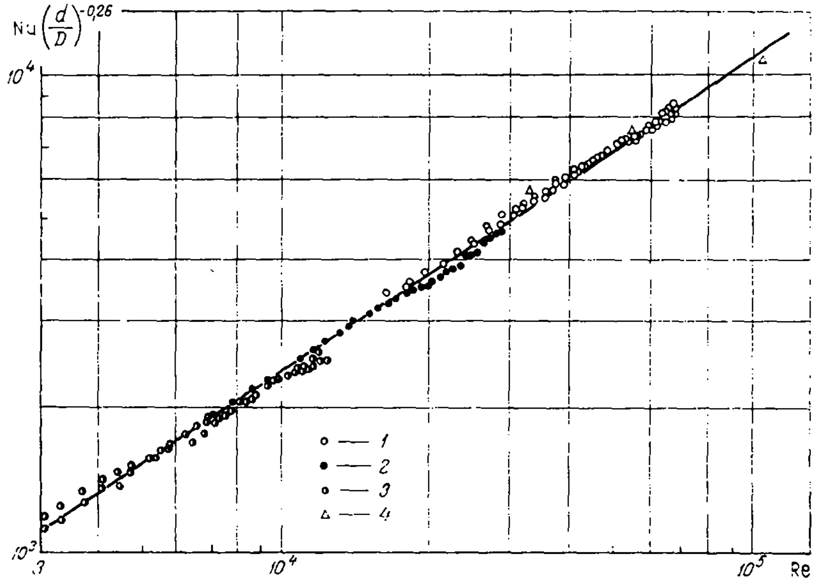

Fig. 4. Universal dependence of the Nusselt number on Re and the ratio d/D at Pr = 7.5: 1) D = 0.3 cm; 2) 0.7; 3) 1.6; 4) 16.

of d/D of 0.025-0.13, and a Prandtl number of 7.5, i.e., for a water salinity of $35^0/_{00}$ and a temperature of 20°C. For other Prandtl numbers the value of the coefficient in (12) will differ from 7.6, but evidently not by more than 30%, since Pr can vary by a maximum of twofold in the ocean while the exponent l, as mentioned above, is close to 0.3-0.4.

Using the approximate form of the temperature dependence of the electrical conductivity [8]

$$\sigma = \sigma_* [1 + b(t - t_*)], \tag{13}$$

with the help of (1), (2), (3), and (12) we obtain an equation for a hydroresistor thermoanemometer in a form analogous to the King law:

$$\frac{I^2 R^2 R_0}{R_0 - R} = BV^{0.63}; \quad B = 68.4 \frac{\pi \lambda A d^{2.26}}{b \sigma_* (\nu D)^{0.63}}. \tag{14}$$

Equation (14) can be used for designing the sensors and electronic circuits of hydroresistor thermoanemometers in the above-indicated ranges of Reynolds numbers and of the ratio d/D. In addition, it allows use to calculate the sensitivity of a thermoanemometer to velocity pulsations for different modes of heating of the sensor. The fact that the numerical coefficient in the expression for B is not determined accurately enough and depends on Pr is not too important for practical purposes, since Eq. (14), like King's equation, makes it possible to bring out the general laws of behavior of the sensitivity, while its concrete values are obtained in the process of calibration.

NOTATION

σ	is the effective electrical conductivity of water in the averaging volume of the sensor, $(\Omega \cdot \mathrm{cm})^{-1}$;
σ_0	is the electrical conductivity of the surrounding medium;
t	is the effective temperature of water heating at sensor, deg;
t_0	is the temperature of surrounding medium;
R	is the active resistance of sensor at the temperature t, Ω;
R_0	is the resistance at the temperature t_0;
I	is the effective value of current through the sensor, A;
F	is the area of heating layer on sensor head, cm^2;
α	is the heat-transfer coefficient;
D	is the diameter of the sensor head;
Nu	is the Nusselt number;
Pr	is the Prandtl number;
Re	is the Reynolds number;

ω is the circular frequency of voltage supplied to sensor, $\sec^{-1}$;
μ is the magnetic permeability of the medium;
v is the local stream velocity beyond the limits of the boundary layer at the surface of the sensor head, cm/sec;
V is the velocity of the undisturbed stream, cm/sec;
x is the distance from center of the microelectrode along the surface of the head, cm;
f is the designation of functional dependence α;
λ is the coefficient of thermal conductivity of water;
ν is the kinematic viscosity;
σ_* is the electrical conductivity of water at the base temperature t_*;
b is the temperature coefficient of electrical conductivity.

LITERATURE CITED

1. A. Okubo, Oceanic Mixing, Mgmt. Info. Serv. (1970).
2. V. S. Belyaev, A. S. Monin, R. V. Ozmidov, and V. T. Paka, "Experimental investigation of fine-scale turbulence in the ocean," Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 10, No. 9 (1974).
3. J. O. Hinze, Turbulence, McGraw-Hill (1959).
4. H. Kramers, Physica, 12, 61 (1946).
5. H. Gröber, S. Erk, and U. Grigull, Principles of the Study of Heat Exchange [Russian translation], IL, Moscow (1958).
6. I. N. Tetel'baum, Electrical Modeling [in Russian], Fizmatgiz, Moscow (1959).
7. L. G. Loitsyanskii, Mechanics of Liquid and Gas, Pergamon (1965).
8. V. V. Skorchelletti, Theoretical Electrochemistry [in Russian], Khimiya, Leningrad (1969).

DESCRIPTION OF OPERATION OF JET GAS–LIQUID PUMP IN THE APPROXIMATION OF A NONEQUILIBRIUM MULTIPHASE-FLOW MODEL

S. D. Frolov

UDC 532.529:533.6

A description of the operation of a jet gas–liquid pump is obtained; the agreement between the results of calculation and experiment is satisfactory.

The jet gas–liquid pump (JGLP), proposed initially as a starting device for liquid metal MHD equipment [2] and then as a rocket-motor fuel pump [3], has not found its expected applications because of its low efficiency. However, as shown by more recent research [13], despite the highly dissipative energy transformations characteristics of all jet apparatus, the JGLP efficiency may be significantly increased.

The optimum organization of JGLP operation in each specific application requires the adequately complete and reliable description of its operation.

The JGLP operation may be represented schematically as follows (Fig. 1).

The quasihomogeneous gas (vapor)–liquid mixture formed in mixer 1 or the liquid heated to saturation point is accelerated in the two-phase nozzle 2 with slight and monitored slip between the phases. The acceleration of the liquid is mainly due to the action of aerodynamic forces from the carrier gas. Not only momentum transfer between the phases but also heat and mass transfer are possible. After the separation of the liquid in the high-velocity separator 4 its kinetic energy is transformed into the potential energy of pressure forces in the diffusor 5. Because of the presence of some of the carrier gas in the separated liquid, the flow in the diffusor is of bubble structure. Phase transformations may also occur in the diffusor.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 5, pp. 834-841, November, 1978. Original article submitted October 24, 1977.

0022-0841/78/3505-1310$07.50